III MODULE

Heat Transfer

Heat transfer is the flow of heat from one object to another. Generally, the difference in the temperature causes a transfer of heat from one part to the other. This transfer can occur by one or more of the three elementary heat transfer processes-conduction, convection and radiation.

Conduction is that mode of heat transfer in which heat is transferred from one part of the system to another by the exchange of internal energy of the molecules or the atoms with their neighbours without causing a net macroscopic flow of mass or material. We can that in conduction the medium is at rest. Convection is defined as that mode of heat transfer in which the transfer takes place trough macroscopic movement of the material from one part of the system to the other. It means that the medium is in motion. In radiation we can say that the mode of transfer is in the form of electromagnetic wave or radiations. This mode of transfer does not need any medium.

CONDUCTION

According to the second law of thermodynamics, heat will flow from any part of a body at a higher temperature to another at lower temperature. We can assume that the heat flux is directly proportional to the temperature gradient which also acts as a driving force for heat flow. Mathematically

$$Q = -K_x A \frac{\partial T}{\partial x} \tag{1}$$

Where Q is the rate of heat transfer per unit time, T is the absolute temperature in kelvin, A the area normal to the direction of flow and K_x is the constant of proportionality known as thermal conductivity, its unit is J/m/s/K in SI. The equation (1) is commonly known as Fourier's law of heat conduction.

HEAT CONDUCTION EQUATION

GENERAL HEAT CONDUCTION EQUATION FOR CARTESIAN

CO-ORDINATES
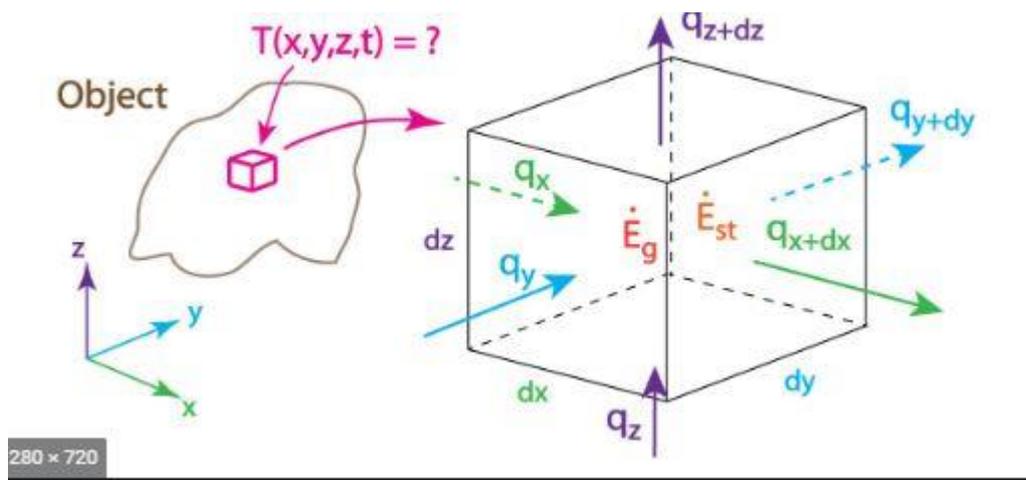
In the above diagram, all the dimension of the cube is given as dx, dy and dz and the volume of the cube is V = dxdydz.

The direction of the heat transfer is shown in the figure and we have to find out the net heat transfer in the system

So to do it we have to find out heat transfer at each dimension

HEAT FLOW IN x-DIRECTION

We know from the Fourier equation that

$Q_x = -K_x[dy.dz]\frac{dT}{dx}.dt$ (2)

where dy.dz is the area perpendicular to the heat transfer $\frac{dT}{dx}$ is the temperature gradient and

dt is the time interval

After some time the heat flow in x-direction will be

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}[Q_x].dx \qquad (3)$$

Net heat flow rate in x-direction

$$dQ_x = Q_x - \{\; Q_x + \frac{\partial}{\partial x}[Q_x].dx\}$$

$$= Q_x - Q_x - \frac{\partial}{\partial x}[Q_x].dx$$

$$= -\frac{\partial}{\partial x}[Q_x].dx$$

$$= -\frac{\partial}{\partial x}[-K_x[dy.dz]\frac{dT}{dx}]dt.dx$$

Keeping aside all the constant term

$$=dy.dz.dt.\frac{\partial}{\partial x}[K_x\frac{dT}{dx}].dx$$

$$=dxdydzdt\frac{\partial}{\partial x}[K_x\frac{dT}{dx}] \qquad (4)$$

Similarly in y and z direction

$$dQ_y = dxdydzdt\frac{\partial}{\partial y}.[K_y\frac{dT}{dy}] \qquad (5)$$

$$dQ_z = dxdydzdt\frac{\partial}{\partial z}.[K_z\frac{dT}{dz}] \qquad (6)$$

so the total amount of heat in the system:-

add 4, 5 and 6

i.e., $\{\frac{\partial}{\partial x}[K_x\frac{dT}{dx}] + \frac{\partial}{\partial y}.[K_y\frac{dT}{dy}] + \frac{\partial}{\partial z}.[K_z\frac{dT}{dz}]\}dxdydz \qquad (7)$

now consider if any heat generation stay in the system, so heat produced by system generator is

$Q_g = q_g *$volume of the system * time

$= q_g.dxdydz.dt$ (8)

HEAT PRODUCED BY SYSTEM

$= mcdT$

$= \rho dxdydz.c.\frac{dT}{dt}.dT$ (9)

Now equating all equation

i.e., heat flow in the system + heat produced by system generator = heat produced by the system

the final equation will be

$$\frac{\partial}{\partial x}\left[K_x\frac{dT}{dx}\right] + \frac{\partial}{\partial y}.\left[K_y\frac{dT}{dy}\right] + \frac{\partial}{\partial z}.\left[K_z\frac{dT}{dz}\right] + q_g = \rho c\frac{dT}{dt}\dots\dots\dots9$$

Here dT = change in temperature

dt = time interval

c = heat capacity

K= conductivity

IF SYSTEM IS ISOTROPIC AND HOMOGENOUS i.e.,Kx=Ky=Kz

The equation will be

$$K\left(\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2}\right) + q_g = \rho c\frac{dT}{dt}$$

$$\left(\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2}\right) + \frac{q_g}{K} = \frac{\rho c}{K}\frac{dT}{dt}\ldots\ldots\ldots\ldots\ldots\text{HEAT CONDUCTION EQUATION(10)}$$

CASES OF HEAT CONDUCTION EQUATION

(i)   Heat conduction without any heat-producing consuming/ reaction

i.e., $q_g = 0$

Therefore the equation will become

$$\frac{dT}{dt} = \alpha\left(\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2}\right)$$

Here α is known as thermal diffusivity which is $\alpha = \frac{K}{\rho c}$ and its unit is m²/s.

(ii)   Heat conduction in steady-state

i.e., $\frac{dT}{dt} = 0$

the equation will become

$$\alpha\left(\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2}\right) = 0$$

or $\alpha\nabla^2\text{T} = 0$

∇^2 is the laplace operator

(a) Heat conduction equation in the cylindrical coordinate system is given by

$$\frac{dT}{dt} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right) + \frac{d^2T}{d\theta^2} + \frac{d^2T}{dz^2}\right]$$

HEAT CONDUCTION THROUGH COMPOSITE WALL

The thermal resistance concept

The Fourier quation for steady conduction through a constant area plane wall, can be written

$$Q_{conduction} = -KA\frac{dT}{dx} = KA\frac{T_1-T_2}{L}$$

This can be re-arranged as:

$$Q_{conduction} = \frac{T_1-T_2}{R_{wall}}$$

$$R_{wall} = \frac{L}{kA}$$

R_{wall} is the *thermal resistance* of the wall against heat conduction or simply the *conduction resistance* of the wall.

The heat transfer across the fluid/solid interface is based on *Newton's law of cooling*

$$Q = hA(T_s - T_\infty)$$

$$R_{Conv} = \frac{1}{hA}$$

Rconv is the thermal resistance of the surface against heat convection or simply the *convection resistance* of the surface.

Thermal radiation between a surface of area *A* at *Ts* and the surroundings at *T∞* can be expressed as:

$$Q = \varepsilon\sigma A(T_s^4 - T_\infty^4) = h_{rad} A (T_s - T_\infty) = \frac{(T1 - T\infty)}{R_{rad}}$$

$$R_{rad} = \frac{1}{h_{rad}A}$$

$$h_{rad} = \varepsilon\sigma(T^2_s + T^2_\infty)(T_s + T_\infty)$$

where $\sigma = 5.67 \times 10^{-8}$ [W/m^{2}K^4] is the Stefan-Boltzman constant. Also $0 < \varepsilon < 1$ is the emissivity of the surface. Note that both the temperature must be in Kelvin.

THERMAL RESISTANCE NETWORK

A composite wall refers to a wall of a several heterogeneous layers, e.g., walls of dwelling houses where bricks are given a layer of plaster on either side. Likewise walls of furnaces, boilers and other heat exchange devices consist of several layers; a layer for mechanical strength or for high temperature characteristics (fire brick), a layer of low thermal conductivity material to restrict the flow of heat (insulating brick) and another layer for structural requirements for good appearance (ordinary brick).

Fig. 2 shows one such composite wall having three layers of different materials tightly fitted to one another. The layers have thicknesses δ_1, δ_2, δ_3 and their thermal conductivities correspond to the average temperature conditions. The surface temperature of the wall are t_1 and t_4 and the temperatures at the interfaces are t_2 and t_3.
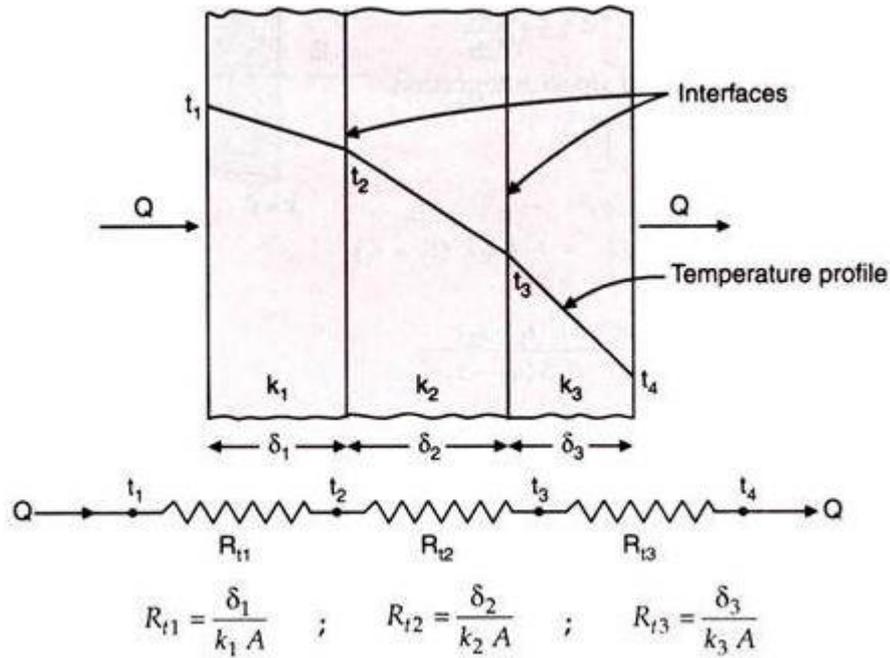
Figure 2: STEADY STATE HEAT CONDUCTION THROUGH A COMPOSITE WALL

Under steady state conditions, heat flow does not vary across the wall, i.e., it is same for every layer. Thus-

$$Q = \frac{k_1 A}{\delta_1}(t_1 - t_2) = \frac{k_2 A}{\delta_2}(t_2 - t_3) = \frac{k_3 A}{\delta_3}(t_3 - t_4)$$

Rewriting the above expression in terms of temperature drop across each layer,

$$t_1 - t_2 = \frac{Q\delta_1}{k_1 A} ; t_2 - t_3 = \frac{Q\delta_2}{k_2 A} ; t_3 - t_4 = \frac{Q\delta_3}{k_3 A}$$

Summation gives the overall temperature difference across the wall

$$t_1 - t_4 = Q\left(\frac{\delta_1}{k_1 A} + \frac{\delta_2}{k_2 A} + \frac{\delta_3}{k_3 A}\right)$$

Then

$$Q = \frac{(t_1 - t_4)}{\delta_1/k_1 A + \delta_2/k_2 A + \delta_3/k_3 A} = \frac{(t_1 - t_4)}{R_{t_1} + R_{t_2} + R_{t_3}}$$

Distributing of temperature in a plane multilayer wall is represented by a polygonal line (Fig. 3.6).

When the above analysis is extended to an n-layer composite wall, one obtains:

$$Q = \frac{(t_1 - t_{n+1})}{\sum_1^n \delta/k A}$$

where $\sum_1^n \delta/kA$ is sum of the thermal resistances of different layers comprising the composite wall.

Analysis of the composite wall assumes that there is a perfect contact between layers and no temperature drop occurs across the interface between materials.

Unsteady state heat conduction

In the previous section, the heat conduction equations were applied to different steady state conditions, where the temperature of a system was only a function of space, not time. However, most metallurgical process involve not only the function of space but also a function of time. Heat conduction in such case is known as unsteady state heat conduction or transient heat conduction.

Lumped analysis

In lumped system analysis, the internal conduction resistance of the heat flow is negligible compared to the convective resistance at the surface. So, the temperature of the body varies with time, but at any given instant, the temperature within the body is uniform and is independent of position i.e, $T = T(\tau)$ only.

Consider a solid body of arbitrary shape, volulme V, mas m, density ρ, surface area A and specific heat C_p as shown in figure below.

At τ = 0, let the temperature throughout the body be uniform at T = T_i. At the instant τ = 0, let the body be suddenly placed in a medium at a temperature of T_∞ as shown in figure.
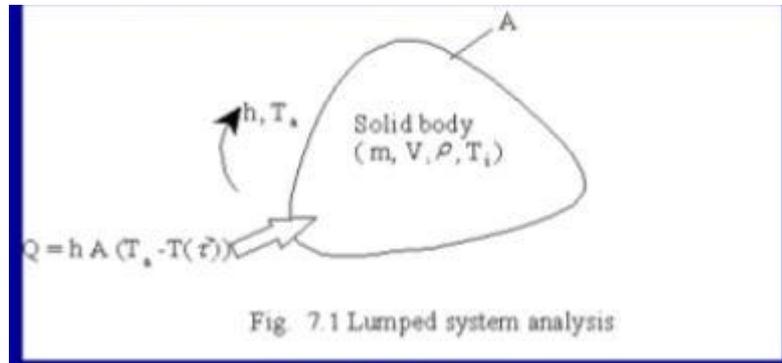
1 lumped system analysis

The energy balance for this situation will be

Amount of heat transferred into the body in time interval dτ = Increase in the internal energy of the body in time interval dτ

So heat enetering into the solid is due to convection and it is expressed as

$Q_{conv} = hA(T_\infty - T(\tau))$

And

$$E = \rho V c \frac{dT}{dt} \tag{a}$$

Introducing the temperature difference θ

$\Theta = T(\tau) - T_\infty$

Differentiating the above equation we get that dT/dt = -dθ/dt, substituting in equation (a) and rearranging the equation we obtain

$$\Theta = \frac{\rho V c}{hA} \frac{d\theta}{dt} \tag{b}$$

By separation of variables and integration from the initial condition at t = 0 and T(0) = T$_i$, we get

$$\frac{\rho V c}{hA} \ln\frac{\theta}{\theta_i} = t \qquad\qquad (c)$$

Where $\theta_i = T_\infty - T_i$

The fraction $\dfrac{\theta}{\theta_i}$ can be obtained by rearranging the above equation such

$$\frac{\theta}{\theta_i} = \frac{T_\infty - T}{T_\infty - T_i} = \exp\left[\left(\frac{hA}{\rho V c}\right) t\right] \qquad\qquad (d)$$

Equation (d) can also be called as Newtonian cooling (or heating). It shows that the temperature falls exponentially with time.

IV MODULE

NATURAL AND FORCED CONVECTION

Convection is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion. Convection is classified as natural and forced convection depending on how fluid motion is initiated . in natural convection any fluid motion is caused by natural means such as buoyant effect, i.e., the rise of warmer fluid and the fall of cooler fluid. Whereas in forced convection, the fluid is forced to flow over a surface or in the tube by external means such as a pump or fan.

Mechanism of forced convection

Convection heat transfer is complicated since it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer (the higher the velocity the higher the heat transfer rate).

The rate of convection heat transfer is expressed by Newton's law of cooling:

$$q = h(T_s - T_\infty)$$

$$Q = hA(T_s - T_\infty)$$

The convective heat transfer coefficient h strongly depends on the fluid properties and the roughness of the solid surface, and the type of the fluid flow.
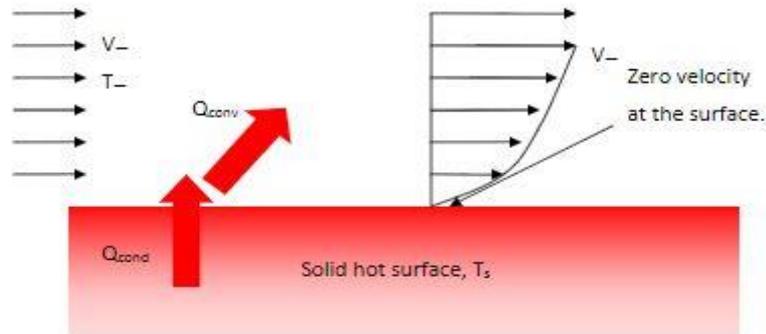
Form the above figure it is assumed that the velocity of the fluid is zero at the wall, this assumption is called no-slip condition. So the heat transfer from the solid surface to the fluid layer is purely conduction since the fluid is motionless. Thus,

$$
\left.\begin{array}{l}
\dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid} \left.\dfrac{\partial T}{\partial y}\right|_{y=0} \\[2mm]
\dot{q}_{conv} = h(T_s - T_\infty)
\end{array}\right\} \rightarrow h = \dfrac{-k_{fluid} \left.\dfrac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}
$$

The convection heat transfer coefficient, in general, varies along the flow direction. The mean or average convection heat transfer coefficient for a surface is determined by averaging the local heat transfer coefficient over the entire surface.

NON-DIMENSIONAL GROUP

The dimensionless group are

Nusselt number: non-dimensional heat transfer coefficient

$$N_u = \frac{h\delta}{k} = \frac{q°_{conv}}{q°_{cond}}$$

Where δ is the characteristic length. Nusselt number represents the enhancement of heat transfer through a fluid as a result of convection relative to conduction across the same fluid layer.

Prandtl number: is a measure of the relative thickness of the velocity and thermal boundary layer

$$P_r = \frac{molecular\ diffusitivity\ of\ momentum}{molecular\ diffusitivity\ of\ heat} = \frac{v}{\alpha} = \frac{\mu C_p}{\alpha}$$

Where α = thermal diffusivity

C_p = specific heat capacity

μ = dynamic viscosity

v = kinematic viscosity

Thermal boundary layer

A thermal boundary layer develops when a fluid at a specific temperature flows over a surface which is at different temperature

2 *thermal boundary layer*
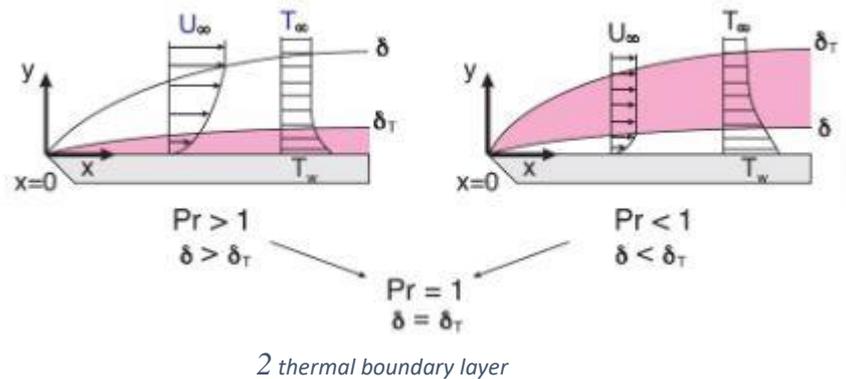
The thickness of the thermal boundary layer δ_T is defined as the distance at which:

$$\frac{T - T_s}{T_\infty - T_s} = 0.99$$

The relative thickness of the velocity and the thermal boundary layer is described by the Prandtl number.

For low Prandtl number fluids, i.e., liquid metals, heat diffuses much faster than momentum flow and the velocity boundary layer is contained within the thermal boundary layer. On the other hand for high Prandtl number like oil, heat diffuses much slower than the momentum and the thermal boundary layer is contained within the velocity boundary layer.

RADIATIVE HEAT TRANSFER

Heat transfer form a body with a high temperature to a body with low temperature, when bodies are not in direct physical contact with each other or when they are separated in space, is called heat radiation. All physical substances in solid, liquid, or gaseous states can emit energy via a process of electromagnetic radiation because of vibrational and rotational movement of their molecules and atoms. Unlike in conduction and convection,, radiative heat transfer doesnot require any medium.

Everything around us constantly emits radiation, and the emissivity represents the emission characteristics of those bodies. This means that every body, including our own, is constantly bombarded by radiation coming from all directions over a range of wavelengths.

Absorptivity (α) is a measure of how much of the radiation is absorbed by the body. Reflectivity(ρ) is a measure of how much is reflected, and transmissivity(τ) is a measure of how much passes through the object.

$$\text{Absorptivity:} \quad \alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{abs}}{G}; \quad 0 \le \alpha \le 1$$

$$\text{Reflectivity:} \quad \rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{ref}}{G}; \quad 0 \le \rho \le 1$$

$$\text{Transmissivity:} \quad \tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{tr}}{G}; \quad 0 \le \tau \le 1$$

BLACKBODY RADIATION

A blackbody is an ideal surface with the following properties.

1. Blackbody radiation absorbs all the incident radiation independent of wavelength and direction.

2. For a given wavelength and temperature, no surface can emit more energy than a blackbody. Planck's distribution of blackbody radiation is given by

$$I_{\lambda,b}(\lambda, T) = \frac{2hc^2}{\lambda^5 \left[e^{\frac{hc}{k_B \lambda T}} - 1 \right]}$$

where $I_{\lambda,B}$ is the blackbody intensity, $h = 6.6252 \times 10^{-34}$ J s, k_B is Boltzmann constant $= 1.38$ 10^{-23} J/K, and c is the speed of light. The blackbody emissive power is given by $e_{b,\lambda}(\lambda,T) = pIb,l(l,T)$. Integration of the blackbody emissive over the entire spectrum gives the famous Stefan-Boltzmann law, $e = \sigma T^4$. σ is the StefaneBoltzmann constant whose value is 5.67×10^{-8} W/(m^2 K) and the unit of temperature is Kelvin.

STEFAN BOLTZMANN LAW

This law states that the rate of radiation energy from the surface per unit area is proportional to the fourth power of the temperature of the body

$$E = \sigma A T^4$$

E = rate of energy emission from the surface

A = surface area of the radiator

σ = Stefan- Boltzmann constant

If we consider a black body with surface temperatures T_1 which radiates to another black body with surface temperature T_2 that completely surrounds it, the second black body completely absorbs the incident energy and emits radiant energy that is proportional to T_2^4. The net heat rate transfer by thermal radiation is then given by:

$$E = \sigma A (T_1^4 - T_2^4)$$

A black body is a perfect radiator. Real bodies, however, do not act like a perfect radiator and emit at a lower rate. To take 108 Nonlinear Systems in Heat Transfer into account the real nature of the radiant bodies, a factor ε, called emissivity, is introduced. Emissivity is defined as the ratio of the emission from a real "gray" surface to the emission from a perfect "black" surface. Then, the rate of radiation heat transfer from a real body at temperature T1 which is surrounded by a black body at temperature T2, is given by:

$$E = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

It is worth pointing out that in most of the practical engineering problems, usually all three-heat transfer mechanisms, namely conduction, convection, and radiation, occur simultaneously.

KIRCHOFF'S LAW

It establishes the relationship between the emissivity and the absorptivity of a body. This relationship may be obtained by studying radiant interchange between two surfaces.

Consider two surfaces, one gray and the other absolutely black. The surfaces are arranged parallel to each other and so close that the radiation of each impinges upon the other. The temperature, emissive power and absorptivity of the surfaces are, respectively, T, E, A and T_0, E_o, A_o; $A_0 = 1$ and $T>T_0$. Let us write the energy balance for the gray surface. A unit area of the gray surfaces emits per unit time a certain amount of energy E [kcal/m^2-hr]. The energy impinging upon the black surface is fully absorbed by it. In its turn, the black surface emits E_0 [kcal/m^2-hr]. A portion AE_0 of this energy is absorbed by the gray body, and the remaining portion, equal to (1-A) E_0, is reflected and fully absorbed by the black surface. Thus, for the gray surface the energy input is AE and expenditure E. Consequently, the balance of radiant interchange is

E_{res}= q= E-AE_0 [kcal/m^2-hr].

Radiant interchange between two surfaces also takes place when $T = T_0$. In this case, the system is in mobile thermal equilibrium and q = 0. Then, from equation we have

$$E = AE_0 \; or \; \frac{E}{A} = E_0$$

The relationship is applicable to any body and therefore may be rewritten as follows:

$$\frac{E_1}{A_1} = \frac{E_2}{A_2} = \frac{E_3}{A_3} = \cdots = \frac{E_0}{A_0} = E_0 = f(T)$$

In this form, Kirchhoff's law is formulated thus**: For all bodies the ratio of the emissive power to absorptivity is the same and is equal to the emissive power of a black body at the same temperature, and depends only on the temperature.**

From Kirchhoff's law it also follows that the emissive power of bodies increases along with their absorptivity. If the absorptivity A of a body is low, its emissive power E is low too. Therefore, good reflectors are poor emitters; for instance, the emissive power of an absolutely white body is zero.

LAMBERT'S LAW

This law states that the total emissive power E_θ from a radiating plane surface in any direction is directly proportional to the cosine of the angle of submission. The angle of emission θ is the angle subtended by the normal to the radiating surface and the direction vector of emission of the receiving surface. If E_n is the total emissive power of the radiating surface in the direction of its normal then

$$E_\theta = E_\theta \cos \theta$$

WEIN'S DISPLACEMENT LAW

The black body radiation curve for different temperature peaks at a wavelength inversely proportional to the temperature

$$\lambda_{maz} \times T = c$$

where T is the absolute temperature in Kelvin

and c is constant
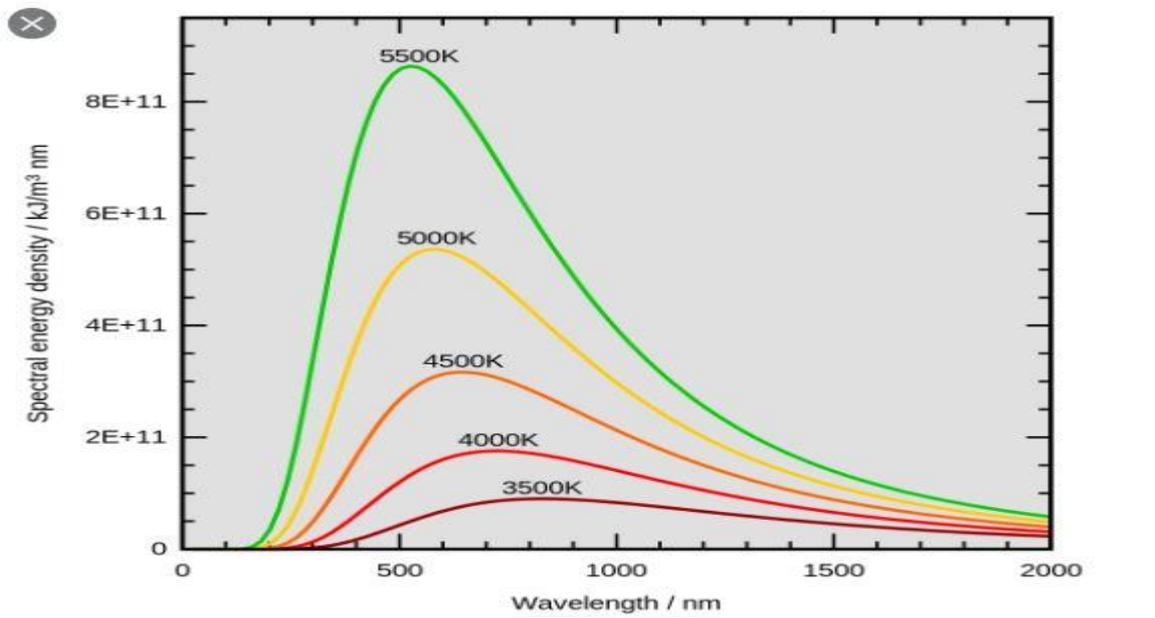
Figure 3: blacbody radiation as a function of wavelength at various wavelenth

GREY BODY

A source with emissivity ($\varepsilon < 1$) independent of wavelength ofteh is referred to as a gray

body. It will have emissive and absorptive power less than that of a blackbody.
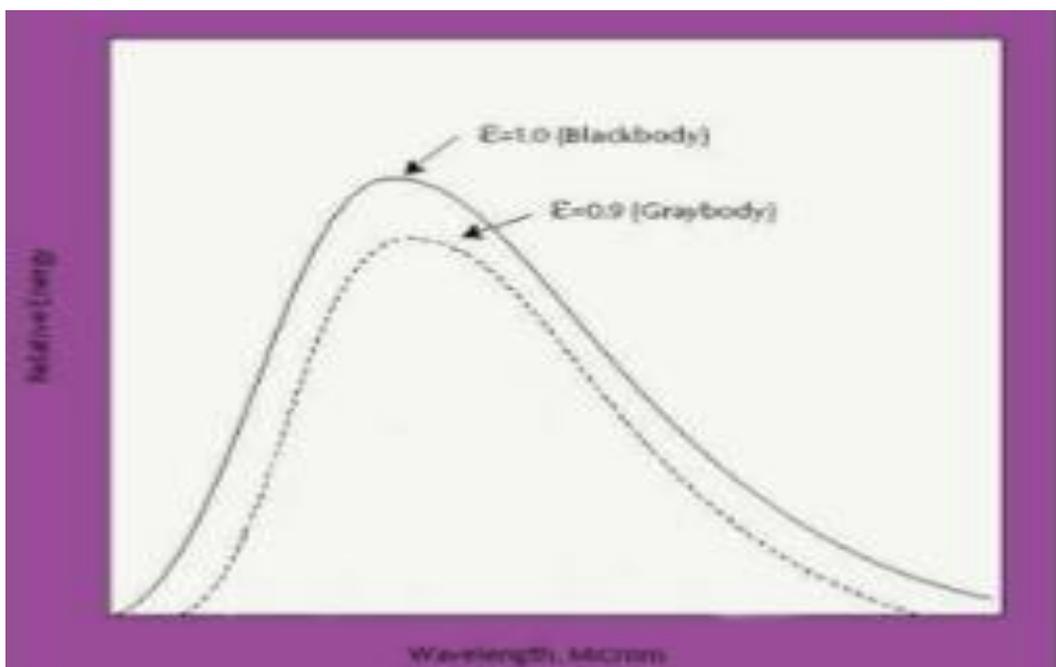
THE VIEW FACTOR

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and temperatures.

It is purely a geometrical parameters that accounts for the effects of orientation on radiation between surface.

In view factor we assume uniform radiation in all directions throughout the surface. Also the surface between two surface doesnot absorb, emit or scatter radiation.

F_{ij} = the fraction of the radiation leaving surface I that strikes the surface j directly
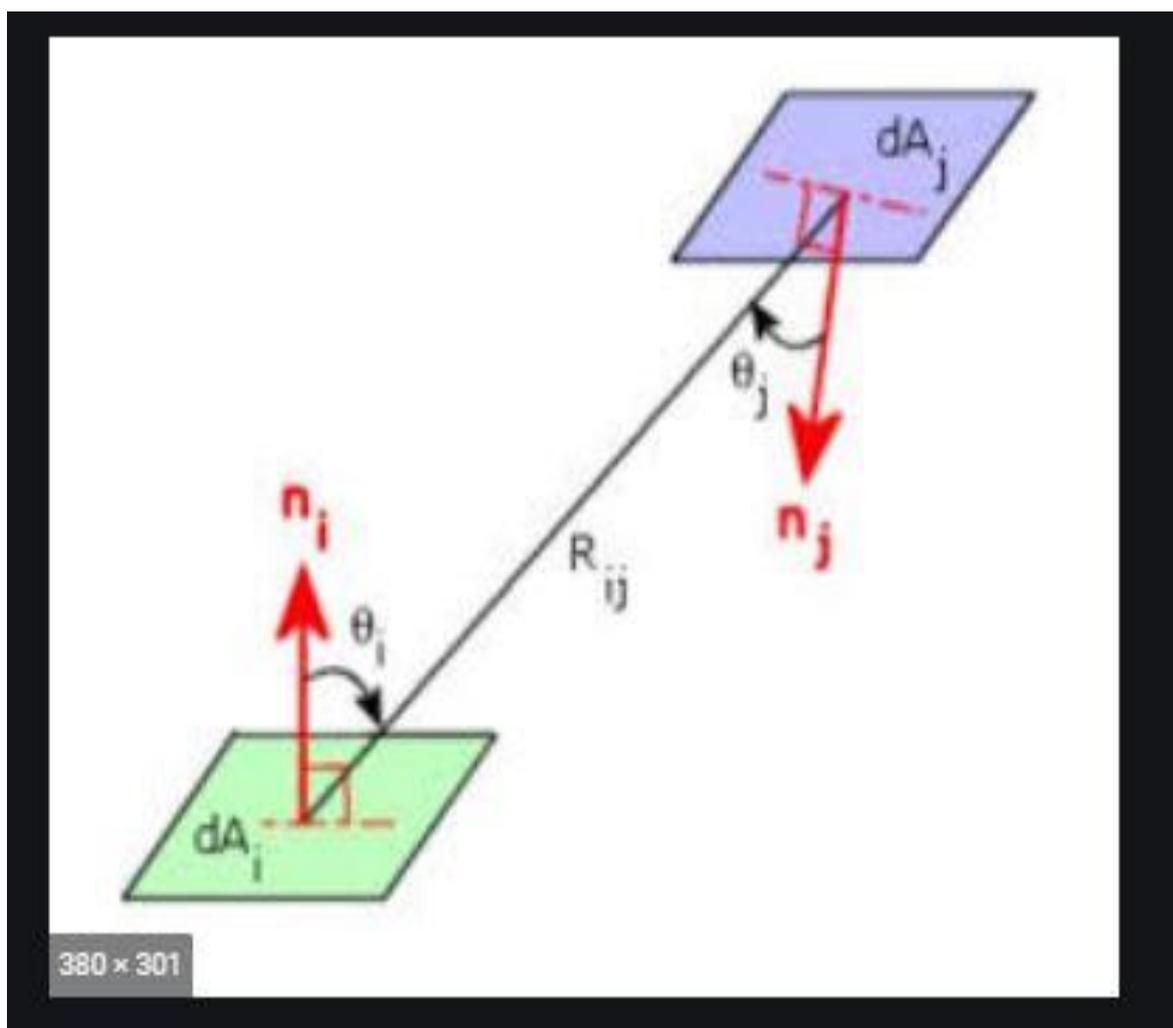
Note the following

- The view factor ranges between zero and one

- $F_{ij} = 0$ indicates that two surfaces donot see each other directly. $F_{ij} = 1$ indicates that the surface j completely surrounds surface i.

- The radiation that strikes the surface doenot need to be absorbed by that surface

- F_{ii} is the fraction of radiation leaving surface i that strikes itself directly. $F_{ii} = 0$ for plane or convex surfaces, and $F_{ii} \neq 0$ for concae surface.

Plane surface,
$F_{ii} = 0$

Convex surface,
$F_{ii} = 0$

Concave surface,
$F_{ii} \neq 0$
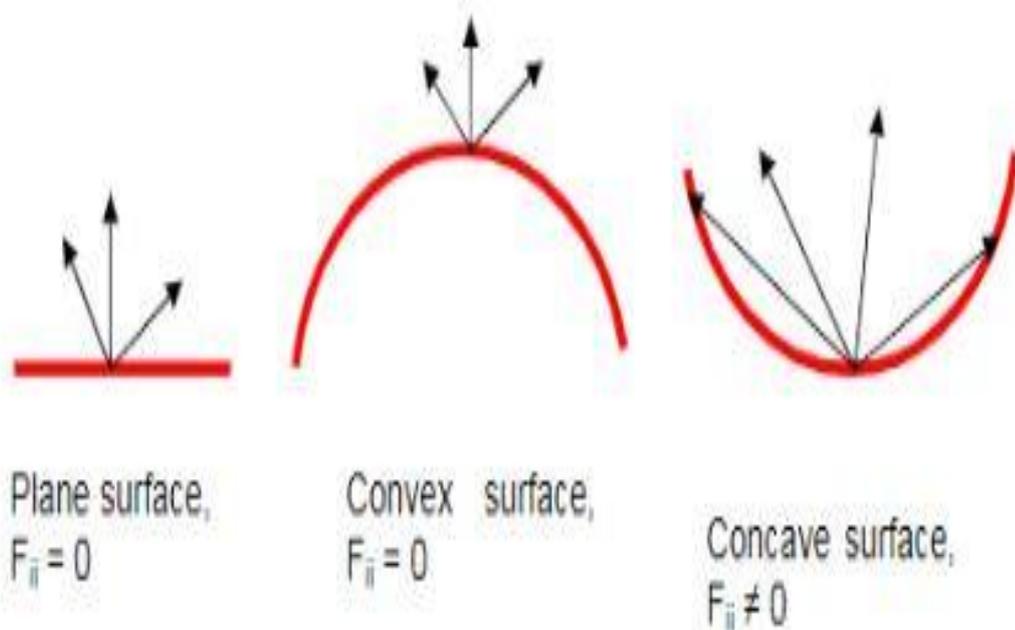
Figure 4: view factor of different surface

The Reciprocity Rule

The view factor F_{ij} is not equal to F_{ji} unless the areas of the two surfaces are equal. It can be shown that:

$$A_i F_{ij} = A_j F_{ji}$$

The Summation Rule

In radiation analysis, we usually form an enclosure. The conservation of energy principle requires that the entire radiation leaving any surface i of an enclosure be intercepted by the surfaces of enclosure. Therefore,

$$\sum_{j=1}^{N} F_{ij} = 1$$

The summation rule can be applied to each surface of an enclosure by varying i from 1 to N (number of surfaces). Thus the summation rule gives N equations. Also reciprocity rule gives 0.5 N (N-1) additional equations. Therefore, the total number of view factors that need to be evaluated directly for an N-surface enclosure becomes

$$N^2 - \left[N + \frac{1}{2} N(N-1) \right] = \frac{1}{2} N(N-1)$$

MODULE V

MASS TRANSFER

It is defined as the transfer of matter by virtue of species concentration difference in a system. The differences in concentration gradient provides the driving force for mass transfer. It always occurs in the direction of reducing concentration gradient.

Mass transfer occurs in two mechanisms

    i.   Diffusion mass transfer

   ii.   Convective mass transfer

    i.   **Diffusion mass transfer**

           In this process the transfer of mass occur by the movement of molecules or species or particles of one component to other. Diffusional mass transfer may occur due to the concentration gradient

   ii.   **Convective mass transfer**

           It is mechanism in which mass is transferred between the fluid and the solid surface as a result od movement of matter from the fluid to the solid surface or fluid.

           Again the convective mass transfer is classified into natural and forced convection mass transfer.

Mass flux

The amount of mass transfer per unit area of the flow is called mass flux.

If m is the amount of mass flow and A is the are normal to the direction of mass flow, then the mass flux $J = m/A$.

Laws of diffusion

Fick's law of diffusion

Fick's first law state that the mass flux J is directly proportional to the concentration gradient of the species.

$$J = -D\frac{\partial c}{\partial x}$$

Where D is the proportionality coefficient called diffusion coefficient or diffusivity.
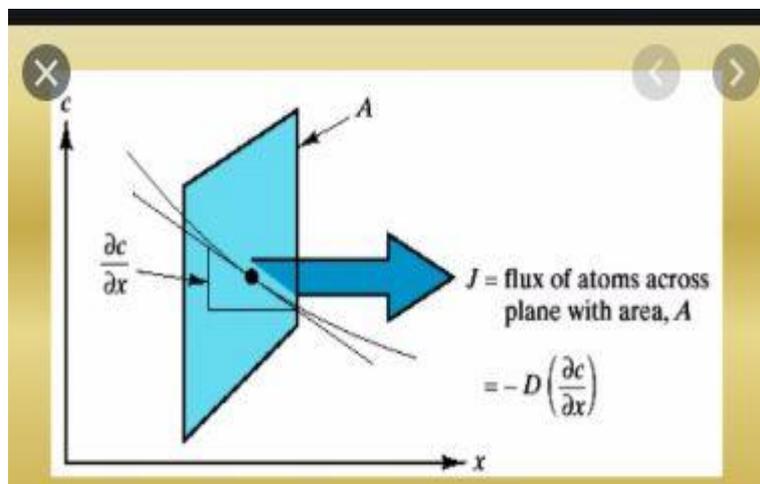
Figure 5: geometry of Fick's first law

The above figure shows that the diffusion is down the concentration gradient that's why the minus sign is in the equation the diffusion driving force is the concentration gradient. This is also known as the steady state equation or similar to the Fourier laws of heat transfer.

Similarly the unsteady state condition is represented by a second order differential equation known as Fick's second law,

$$\frac{\partial c_x}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial c_x}{\partial x}\right)$$

For many practical problem one can assume that D is independent of c leading to a simplified version of the above equation\

$$\frac{\partial c_x}{\partial t} = \left(D \frac{\partial^2 c_x}{\partial x^2} \right)$$

The common application of the above equation is the diffusion of material into a semi-infinite solid while the surface concentration of the diffusion species, c_s remains constant. Two examples of this system would be the plating of metals and the saturation of materials with atmospheric gases. Specifically, the steel surface are often hardened by carburization, the diffusion of carbon atoms into the steel from a carbon-rich environment. The solution to theis differential equation with the given boundary condition is

$$\frac{c_x - c_0}{c_s - c_0} = 1 - erf\left(\frac{x}{2\sqrt{Dt}} \right)$$

Where c_0 is the initial bulk concentration of the diffusing species and erf refers to the Gaussian error function.
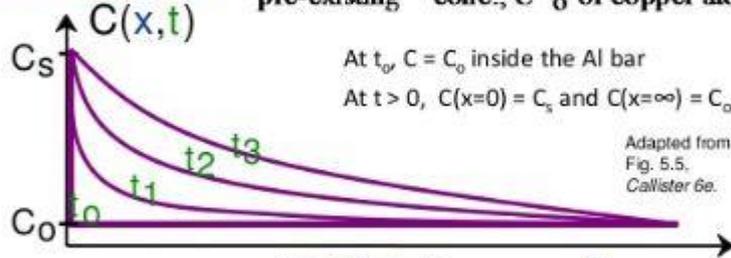
Figure 6: solution of Fick's second law

CONCENTRATION BOUNDARY LAYER

In a flowing system, there will be a relative transport of species and species conservation must be satisfied at each point in the concentration boundary layer. The pertinent form of the conservation equation may be obtained by identifying the processes that affect the **transport** and **generation** of species A for a differential control volume in the boundary layer.

[Derivation is similar to the thermal boundary layer equation; - carry out. Hints: Species A may be transported by advection **(with the mean velocity of the mixture)** and by diffusion **(relative to the mean motion)** in each of coordinate directions.]**

The rate at which the mass of species **A** is generated per unit volume due to such reactions as $\dot{n}_A$. If the total mass density ρ is assumed to be constant we get

$$u\frac{\partial\rho_A}{\partial x} + v\frac{\partial\rho_A}{\partial y} = \frac{\partial}{\partial x}\left(D_{AB}\frac{\partial\rho_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{AB}\frac{\partial\rho_A}{\partial y}\right) + \dot{n}_A \quad \text{...........................1}$$

For mass transfer problems, v can no longer be zero at the surface. However, it will be reasonable to assume $v = 0$ which is equivalent to assuming that mass transfer has a negligible effect on the velocity boundary layer. We note that with mass transfer, the boundary layer fluid is a binary mixture of species A and B In all problems of interest $C_A \ll C_B$ and it is reasonable to assume that the boundary layer properties $(k, \mu, C_p \ etc)$ are those of species C_B

because boundary layer thickness is typically small, the following inequality will apply

$$\frac{\partial C_A}{\partial y} >> \frac{\partial C_A}{\partial x}$$

Using the following non-dimensional variables.

$X = x/L, Y = y/L, U = u/\mho_{av}, V = v/U_{av}$

and $C_A^* = \dfrac{C_A - C_{A,W}}{C_{A,\infty} - C_{A,W}}$

The above equation will become

$$U\frac{\partial C_A^*}{\partial X} + V\frac{\partial C_A^*}{\partial Y} = \frac{1}{Re_L S_c}\frac{\partial^2 C_A}{\partial y^2} \quad \text{.......................................3}$$

where , S_c is Schmidt Number $= nu/D_{AB}$

S_c is the ratio of momentum and mass diffusivities. For mass transfer in a gas flow over an evaporating liquid or sublimating solid, the convection mass transfer coefficient h_m depends on $\quad L, D_{AB}, \rho, \mu$ and U_{av}.

$$C_A^* = f(X, Y, Re_L, S_c)$$

Again,

$$h_m = -\frac{D_{AB}}{L} \frac{(C_{A,\infty} - C_{A,w})}{(C_{A,w} - C_{A,\infty})} \left. \frac{\partial C_A^*}{\partial y} \right|_{y=0}$$

$$h_m = \frac{D_{AB}}{L} \left. \frac{\partial C_A^*}{\partial y} \right|_{y=0}$$

Which is dimensionless concentration gradient at the surface

Sh = Sherwood number $= \frac{h_m L}{D_{AB}} = \left. \frac{\partial C_A^*}{\partial y} \right|_{y=0}$

We have already seen that the significance of **Prandtl number**

$$P_r = \frac{molecular\ diffusitivity\ of\ momentum}{molecular\ diffusitivity\ of\ heat} = \frac{v}{\alpha} = \frac{\mu C_p}{\alpha}$$

Similarly, the **Schmidt number** provides a relative measure of the relative effectiveness

of

momentum and mass transport by diffusion in the velocity and concentration

boundary layers, respectively.

$$Sc = \frac{v}{D} = \frac{\mu}{\rho D} = \frac{viscous\ diffusion\ rate}{molecular\ (mass)\ diffusion\ rate}$$

Another parameter which related to Pr and Sc is the lewis number. It is defined as

$$Le = \frac{\alpha}{D_{AB}} = \frac{Sc}{Pr}$$

INTERFACIAL MASS TRANSFER

In the previous section, we deal with mass transfer only with single phase. However, in metallurgical application mass transfer are concerned with transfer across the phase boundary which may be called as the interfacial mass transfer. Some theories have been suggested to calculate the mass transfer rate across the phase boundary

Two-film theory

First proposed by Lewis and Whitman, they attempt to explain the mass transfer across an interface. In this theory, they assumed that the turbulence dies at the interface and that a laminar layer is established across the interface. Outside these laminar layer, the composition of these phase assumes to be in uniform
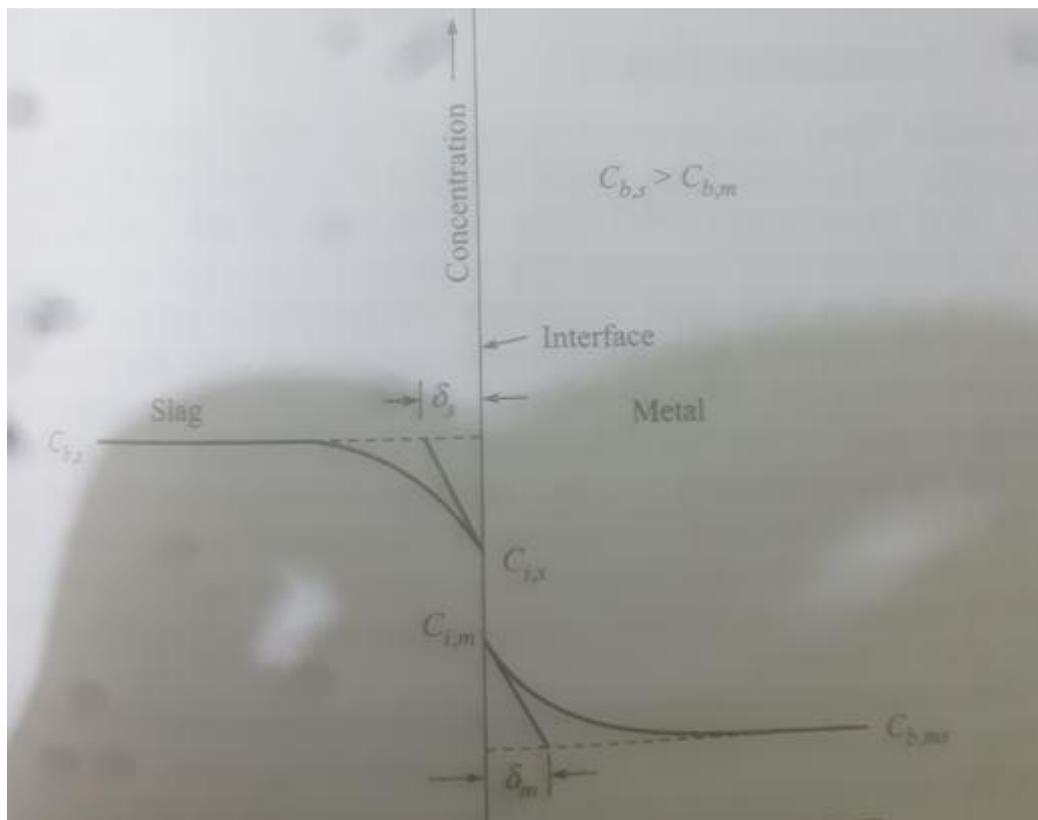
Figure 7: two film theory transfer of a componet from slag phase to metalphase

Are the

In figure 3 the diffusing component is transferred from slag to metal phase. Further, the linear variation in composition implies that the diffusion is under steady-state condition. The rate of transfer J_A of the component A per unit area per unit time is expressed as

$$J_A = \frac{D_s}{\delta_s} \left(C_{b,s} - C_{i,s} \right) = - \frac{D_m}{\delta_m} \left(C_{i,m} - C_{b,m} \right)$$

Where δ_m and δ_s are the boundary layer thicknesses in slag and metal phase respectively. The symbols $C_{b,s}$ and $C_{i,s}$ refers to the concentration in bulk and at the interface in the slag phase and $C_{b,m}$ and $C_{i,m}$ refers to the corresponding bulk and interfacial concentration in metal phase.